

A Method for Modelling Peak Signal Statistics on a Mobile Satellite Transponder

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ABSTRACT

This paper proposes a simulation method to model the peak duration and energy content of signal peaks in a mobile communication satellite operating in an FDMA mode and presents an estimate of those power peaks for a system where the channels are modeled as band limited gaussian noise, which is taken as a reasonable representation for ACSSB, MSK or PSK modulated signals. The simulation results show that, under this hypothesis, the level of the signal power peaks for 10%, 1% and 0.1% of the time are well described by a Rayleigh law and that their duration is extremely short and inversely proportional to the total FDM system bandwidth.

INTRODUCTION

Assuming that the transmitting power in a mobile communication satellite is to be provided by power amplifiers whose instantaneous power consumption is related to the instantaneous T_X power output, it is of interest to specify the peak transmit power levels which will be required and the duration of those peaks. More specifically if the spacecraft power subsystem is sized to provide power up to the 99% point of the average power distribution, it is important to estimate this value, and to estimate the statistical properties of voltage addition peaks at higher probability levels (i.e.: 99.9%) in order to optimize the short term characteristics of the power amplifier response.

If the channels on the satellite transponder are to be ACSSB, MSK or PSK

modulated, this question may be viewed somewhat differently than in the case where the satellite system is required to transmit a large number of narrowband modulated carriers. The individual channels may then be considered to some extent as band-limited gaussian noise.

This paper studies then this question of peak signal statistics on a transponder where the individual channels can be modeled as band-limited gaussian noise. This is analysed for a general transmission system configuration of one thousand voice-activated 5 kHz bandwidth channels with an SCPC-FDMA configuration, which are modeled as 400 fully activated channels randomly chosen inside the FDM band.

THEORETICAL FOUNDATIONS

The theoretical foundations for such an analysis have been developed in the classical papers by S.O. Rice on "Mathematical Analysis of Random Noise", [1,2] In particular, Part III of this ensemble of two papers discusses the statistics of random noise currents: *Probability distributions associated with the maxima of the currents and the maxima of its envelope are developed. Formulas for the expected number of zeros and maxima per second are given...* The results have been presented and expanded in a number of classical textbooks, such as the one by J.S. Bendat [3], where one finds in chapter 3 a discussion of the applications of probability theory to random noise analysis: expected number of maxima per unit of time, distribution of the envelope.

Random noise can be regarded as the sum of a large number of independent events and hence can be related to a gaussian statistical law. The case of noise characterized by particular spectral properties (such as band-limiting) can be studied through the use of its power spectrum and correlation function, which results in gaussian-distributed signal models. The probability of these signals exceeding a certain peak can be easily estimated, and the relationships for signal power peaks can be derived. It is also possible to show that the signal envelope will follow a Rayleigh distribution.

These results permit then to compute the probability that a signal exceeds a certain power level, as is the concern here.

EXPECTED NUMBER OF LEVEL CROSSINGS

Let us consider the random noise $y(t)$ and its time derivative $y'(t)$. Their joint probability expresses the probability that $y(t)$ lies in the interval $(y, y+dy)$ when its derivative is in the interval $y', y'+dy$. To find the expected number of crossings of $y(t)$ through the interval $(y, y+dy)$, the amount of time that $y(t)$ is in the interval must be divided by the time required to cross the interval, which is related $y'(t)$ to the derivative of $y(t)$. Hence, the expected number of passages per unit of time of $y(t)$ through the interval $y'(t) = y'$ for all values of y is given by.

$$N(y(t) = y) = \int_{-\infty}^{+\infty} |y'| f(y, y') dy' \quad (1)$$

where the absolute value of y' is used since crossing time must be a positive quantity. So, if $y = 0$, one will obtain the expected number of zeros. For other values of y , one will obtain the expected number of passages through given levels.

This is directly applicable to estimating the probability of a signal having peaks over a certain power level as is the concern here. One field of application of this

concept has been in the statistical analysis of fades in the mobile radio-channel. Lee [4] (chapters 2 and 5) gives results for the probability density for normal-distributed signal levels in dB.

EXPECTED DURATIONS OF MAXIMA AND MINIMA

Rice [1] had developed the basis on which to estimate the durations of maxima and minima (peaks and fades). This question appears, however, to have been developed further by researchers in the mobile radio field, who have applied it to the estimation of fade durations. It can be as easily applied to the estimation of peak durations.

According to Lee [4] (chapters 2 and 5), for a signal, the relation between the average duration of the signal fade (or of the signal peak) and the expected number of passages per second at a particular level y is given as the ratio between the expected amount of time per second where $y(t)$ is below (or above) the given level y and the average duration of fades under (or of peaks above) this given level.

Results are given by Lee [4] (page 73) for average durations of fades under a given level in the case of a Rayleigh distributed variable, whose time derivative is gaussian distributed. These results could be adapted to the estimation of peak durations, as is the concern here.

ADVANTAGES OF A SIMULATION APPROACH

It has been shown in the previous section that a good theoretical foundation has been established for the understanding and the analysis of the question under concern here, namely the modeling of the peak duration statistics for band-limited noise channels in an FDM configuration.

From this theoretical foundation, one expects the satellite channel signal to be normal-distributed. One expects its envelope to be Rayleigh-distributed. One expects then the probability of the power peaks to follow

those of a Rayleigh distributed signal. As illustrated, for example in Lee [4], page 175, the 90%, 99%, 99.9% power levels would be about 3,7 or 9 dB above the signal rms value 10% 1% or 0.1% of the time.

One would then have to make the required assumptions on the signal statistics and on its power spectra to undertake the analytical derivations or the numerical integrations leading to the estimates of the frequency of occurrence of these power peaks and to the estimates of their durations.

It appears then legitimate to ask a number of questions about the quantitative aspects of the model. Let us admit the mobile satellite channel is normal-distributed with a power spectrum and a certain variance. But one needs also to make assumptions about the power spectrum and the variance of the derivative in order to compute the joint probability density function $f(y,y')$. At a certain point, the level of confidence in the quantitative results becomes limited. Quoting from J.S. Bendat [3], page 128, it can be written that:

"In applying those formulas, many assumptions about the physical nature of $y(t)$ are involved so that it is quite gratifying to learn, when one is through, that good agreement with empirical data has been found."

So, for the completion of this study, and in order to obtain with a high degree of confidence the quantitative results desired on the peak traffic statistics, it was judged preferable to proceed to the simulation of the FDM channel, or rather of a restricted subset of the channel.

SIMULATION

As mentioned earlier the general transmission system configuration was one with one thousand voice-activated 5 kHz bandwidth channels with an SCPC-FDMA configuration, modeled as 400 fully activated channels randomly chosen inside this 5 MHz FDM band.

The simulation was carried out on a SUN workstation. For consideration related to the speed and capacity of the work station, it was found impractical to simulate the FDM system mentioned above (400 channels out of 1000 in a 5 MHz bandwidth). Simulations were then carried out for 4 channels out of 10 in a 50 kHz bandwidth, and for 40 channels out of 100 in a 500 kHz bandwidth: the results were then extrapolated for the case of 400 channels out of 1000 in a 5 MHz bandwidth.

To realize the individual 5 kHz channel simulation, a gaussian noise over a 10 kHz bandwidth was generated and convolved with a 3 kHz 3 dB bandwidth baseband filter having a 30 dB attenuation outside the 5 kHz band.

SIMULATION WITH BASEBAND CHANNELS

The simulation procedure was first tested for the case where forty 5 kHz baseband channels are summed. With the signal amplitude normalized for a power of unity, the power probability distribution and the density function were computed. From these, power levels of 1.9, 6.9 and 8.7 db respectively over the rms value for 10%, 1% and 0.1% are measured or calculated directly from the simulated signal: these values are well in agreement with a Rayleigh model (Lee [4], p. 175). The frequency of occurrence (number/second) of these power peaks is 185.6, 31.7 and 2.4 per second.

SIMULATION WITH 5 KHZ CHANNELS IN FDM CONFIGURATION

As stated in section 4, the simulation took into account a maximum number of forty out of one hundred 5 kHz bandpass channels, which was a compromise between the calculation time and the precision of the results. In order to estimate the results for four hundred out of one thousand bandpass channels in an FDM configuration, the relation between the bandwidth and the statistical parameters had to be obtained. The results for four out of ten 5 kHz bandpass channels and for forty out of one hundred 5

kHz bandpass channels were computed, and extrapolated.

To realize this simulation, a gaussian noise over an appropriate bandwidth was generated and convolved with a bandpass filter of 3 kHz bandwidth centered at a given frequency f_c . This operation was repeated for different values of f_c . The next step was to sum these filtered signals to obtain the total signal.

RESULTS AND CONCLUSION

As a result of these simulations, and of the extrapolation to the case of four hundred active 5 kHz channels out of one thousand in an FDM configuration, the following results have been obtained for the signal level distribution, the frequency of occurrence and the duration of power peaks.

The signal level distribution is well described by a Rayleigh law and the signal levels in dB, above the signal rms value, for the 90%, 99% and 99.9% probability levels are in the order of:

90%: 1.6 dB above the signal rms value
99% : 6.9 dB above the signal rms value
99.9% : 9.1 dB above the signal rms value

In other words, the instantaneous signal power will be 1.6 dB above the rms value 10% of the time, 6.9 dB above the rms value 1% of the time and 9.1 dB above the rms value 0.1% of the time.

For the frequency of occurrence (per second) of the peaks above the 90%, 99% and 99.9% probability level, it is estimated to be in the order of:

90%: (1.6 dB): 400 000 per second
99%: (6.9 dB): 49 000 per second
99.9%: (9.1 dB): 4 900 per second

In other words, the instantaneous power level will rise 1.6 dB above the rms value 400 000 times/second, will rise 6.9 dB above the rms value 49 000 times per second and will rise 9.1 dB above the rms value 4 900 times per second.

As for the duration of those peaks, they are extremely short. 90% of the peaks which exceed the signal rms value by 1.6 dB, 6.9 dB or 9.2 dB will have a duration not exceeding 0.28, usec, 0.18 us or 0.19 usec respectively

As a conclusion, it may be said that simulation is an efficient and quick method to obtain information on the statistical power structure of a signal in a satellite transponder. One basic issue however is to have a valid statistical representation of the signal carried by the individual channels. To that extent, it would be important for future work to include such questions as a more accurate modeling of the user channel signal for specific types of modulation such as ACSSB or QPSK, taking into account the problem of possible addition of discrete spectral components present in the individual signals.

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